

General Certificate of Education
June 2009
Advanced Subsidiary Examination



MATHEMATICS
Unit Further Pure 1

MFP1

Monday 1 June 2009 9.00 am to 10.30 am

For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The equation

$$2x^2 + x - 8 = 0$$

has roots α and β .

- (a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*
- (b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*
- (c) Find a quadratic equation which has roots $4\alpha^2$ and $4\beta^2$. Give your answer in the form $x^2 + px + q = 0$, where p and q are integers. *(3 marks)*

2 A curve has equation

$$y = x^2 - 6x + 5$$

The points A and B on the curve have x -coordinates 2 and $2 + h$ respectively.

- (a) Find, in terms of h , the gradient of the line AB , giving your answer in its simplest form. *(5 marks)*
- (b) Explain how the result of part (a) can be used to find the gradient of the curve at A . State the value of this gradient. *(3 marks)*

3 The complex number z is defined by

$$z = x + 2i$$

where x is real.

- (a) Find, in terms of x , the real and imaginary parts of:
- (i) z^2 ; *(3 marks)*
- (ii) $z^2 + 2z^*$. *(2 marks)*
- (b) Show that there is exactly one value of x for which $z^2 + 2z^*$ is real. *(2 marks)*

4 The variables x and y are known to be related by an equation of the form

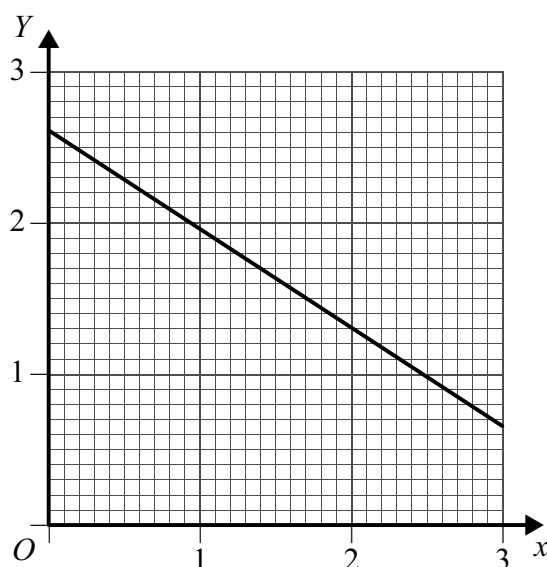
$$y = ab^x$$

where a and b are constants.

(a) Given that $Y = \log_{10} y$, show that x and Y must satisfy an equation of the form

$$Y = mx + c \quad (3 \text{ marks})$$

(b) The diagram shows the linear graph which has equation $Y = mx + c$.



Use this graph to calculate:

- (i) an approximate value of y when $x = 2.3$, giving your answer to one decimal place;
- (ii) an approximate value of x when $y = 80$, giving your answer to one decimal place.

(You are not required to find the values of m and c .) (4 marks)

5 (a) Find the general solution of the equation

$$\cos(3x - \pi) = \frac{1}{2}$$

giving your answer in terms of π . (6 marks)

(b) From your general solution, find all the solutions of the equation which lie between 10π and 11π . (3 marks)

Turn over ►

6 An ellipse E has equation

$$\frac{x^2}{3} + \frac{y^2}{4} = 1$$

(a) Sketch the ellipse E , showing the coordinates of the points of intersection of the ellipse with the coordinate axes. (3 marks)

(b) The ellipse E is stretched with scale factor 2 parallel to the y -axis.

Find and simplify the equation of the curve after the stretch. (3 marks)

(c) The **original** ellipse, E , is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$. The equation of the translated ellipse is

$$4x^2 + 3y^2 - 8x + 6y = 5$$

Find the values of a and b . (5 marks)

7 (a) Using surd forms where appropriate, find the matrix which represents:

(i) a rotation about the origin through 30° anticlockwise; (2 marks)

(ii) a reflection in the line $y = \frac{1}{\sqrt{3}}x$. (2 marks)

(b) The matrix \mathbf{A} , where

$$\mathbf{A} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. (2 marks)

(c) The transformation represented by \mathbf{A} is followed by the transformation represented by \mathbf{B} , where

$$\mathbf{B} = \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$$

Find the matrix of the combined transformation and give a full geometrical description of this combined transformation. (5 marks)

8 A curve has equation

$$y = \frac{x^2}{(x-1)(x-5)}$$

- (a) Write down the equations of the three asymptotes to the curve. *(3 marks)*
- (b) Show that the curve has no point of intersection with the line $y = -1$. *(3 marks)*
- (c) (i) Show that, if the curve intersects the line $y = k$, then the x -coordinates of the points of intersection must satisfy the equation

$$(k-1)x^2 - 6kx + 5k = 0 \quad \text{span style="float: right;">*(2 marks)*$$

- (ii) Show that, if this equation has equal roots, then

$$k(4k+5) = 0 \quad \text{span style="float: right;">*(2 marks)*$$

- (d) Hence find the coordinates of the two stationary points on the curve. *(5 marks)*

END OF QUESTIONS

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